Multi-goal Planning with Non-Linear Programming Model

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***Abstract*— The herein-studied multi-goal planning is motivated by the visual inspection tasks using a robotic manipulator on a mobile platform. A camera is attached to the manipulator to take snapshots of objects of interest from predefined 3D regions. The problem can be formulated as the Generalized Traveling Salesman Problem with Neighborhoods (GTSPN), and the lower and upper bounds on the optimal solution can be obtained by the Branch-and-Bound (BnB) method. However, it is necessary to address the motion constraints of the robot. We propose to formulate the GTSPN with the robot's kinematic constraints as the Non-Linear Program (NLP) model employed in the BnB to determine bounds of the optimal solution. Although the global optimality guarantee of the particular subproblems relies on the NLP solver, the preliminary results with the `Ipopt` solver yield feasible solutions, supporting the proposed idea of using the NLP model in robotics routing.**

I. Introduction

In this work-in-progress report, we present robotics routing as a variant of multi-goal planning motivated by a visual inspection task with a mobile manipulator. The task is to capture objects of interest from all the predefined target regions. The considered mobile platform is a robotic arm on a multi-legged walking robot that can move almost freely with the arm and its body within a certain neighborhood of its current stance, thus allowing it to exploit additional degrees of freedom to efficiently address the visual inspection tasks.

The problem is determining a cost-efficient visual inspection path to visit the required target regions. The planning problem can be formulated as the *Traveling Salesman Problem with Neighborhoods* (TSPN), in which the optimal sequence of visits to the regions is determined with the exact locations of visits to the regions. Note that the sequencing part of the TSPN is also known as the robotic task sequencing [1], highlighting robotic constraints in routing.

For visual inspection tasks [2] (or data collection in general [3]), we can exploit the regions for additional degrees of freedom to determine suitable locations of visits to the regions. Thus, the travel cost can be saved compared to solving an instance of the purely combinatorial TSP [4]. However, regions can be of various shapes depending on the task and can be relatively complex to describe and characterize. Therefore, we consider a generalization of the TSPN where a particular target is a set of possibly overlapping convex regions. The problem is called the *Generalized TSPN* (GTSPN) [5], and it stands to determine a cost-efficient path that visits at least one region within each set.

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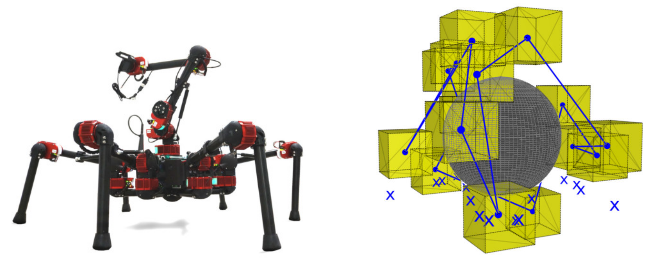

(a) Six-legged walking robot with a robotic arm and attached camera.

(b) The GTSPN instance with $n = 15$ regions and its solution.

Fig. 1. Mobile platform with robotic arm utilized in the experimental validation of the proposed approach, and an instance of the problem.

A solution to the GTSPN with convex regions can be found using *Branch-and-Bound* (BnB) [6]. However, it is desirable to address the motion constraints of the mobile manipulator to find a feasible multi-goal path directly. Disregarding the constraints during the sequence determination might lead to an unfeasible sequence. Therefore, the constraints are introduced to the BnB, and lower bound values on the optimal solution cost are determined—assuming the mobile platform can move in any direction in a plane with the possibility to adjust the body's height by the legs' joints.

The considered walking mobile manipulator (see Fig. 1a) is modeled as a system with 7 *Degree of Freedom* (DoF), where 3 DoF represent the absolute position of the robot body and 5 DoF describe the joint coordinates of the arm with the camera in the wrist. We assume that the body can reach any position by a sequence of steps [7]. Thus, the challenge is to satisfy the robot's motion constraints in the solution of the GTSPN. The proposed idea is to utilize the *Non-Linear Programming* (NLP) to model the robot's forward kinematics and motion constraints and, subsequently, use the NLP model to solve BnB subproblems. Unlike in [8], [9], where mathematical models are employed in determining trajectories, the proposed NLP model is employed to determine feasible configurations of the robot that satisfy the inspection task constraints and the robot's kinematics constraints. The configurations to visit the given sequence of regions are determined to minimize the travel cost (time) of the multi-goal path.

The approach (to the best of the authors' knowledge) is the first BnB-based solution to multi-goal planning with a mobile manipulator; see a solution example depicted in Fig. 1b. The approach has been validated with a single region within each set of the GTSPN and using the `Ipopt` solver [10] that provides only locally optimal solutions. Although the optimality guarantee of the proposed approach relies on the optimal solution of the NLP model, and thus the capabilities of the employed solver, found solutions support the viability of the proposed BnB-based solution with competitive solutions cost to the discretized instances of the GTSPN.

II. BnB-based Multi-goal Planning

The proposed approach follows the BnB to the *Close Enough TSP* [11], where the solution space is a tree, where each tree node contains a partial problem, defined as a sub-sequence of regions, with a partial solution, which value is a problem lower bound. The problem's upper bound can be computed as the feasible solution to the partial solution by inserting not covered regions into the partial solution, thus creating a feasible solution. In the studied GTSPN, each partial solution is a solution of the corresponding NLP model.

The main challenge is to satisfy the robot's motion constraints in determining the multi-goal path cost for a particular sub-sequence using the robot's travel cost from one inspection configuration to the following configuration of the next region in the sub-sequence. Moreover, we need to determine the robot's configurations such that the camera is within the particular region and pointed towards the corresponding object of interest to satisfy inspection task constraints on taking snapshots of the object. The configurations are determined by the proposed NLP model for a sub-sequence of regions using generalized kinematics.

A. Preliminary Results

The feasibility of the proposed BnB-based approach with the NLP model has been empirically evaluated for inspection scenarios to visit 3D regions with the defined required direction of the camera to point in the desired direction toward objects of interest. Since the existing global NLP solvers are incapable of solving the proposed NLP model without significant approximations, we considered two variants of the proposed BnB-based solver.

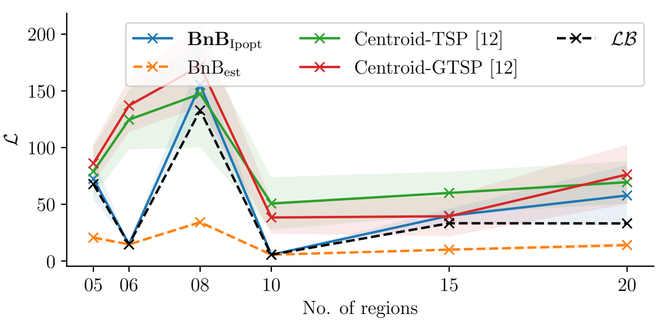

Fig. 2. Solution costs $\mathcal{L}$ of 3D GTSPN instances aggregated by number of regions. Lower bound values $\mathcal{LB}$ are obtained by the $\mathrm{BnB}_{\mathrm{Ipopt}}$ solver.

First, `Ipopt` solver [10] is employed in $\mathrm{BnB}_{\mathrm{Ipopt}}$, without the guarantee that the optimal solutions of the partial problems are found; however, the method still provides locally optimal feasible solutions. Second, the weighted Euclidean distance of the robot's body and end-effector position is used to estimate the lower bounds in $\mathrm{BnB}_{\mathrm{est}}$ disregarding forward kinematic constraints, providing weak lower bounds. An insertion heuristic is employed for both BnB-based solvers to determine a feasible solution. Besides, we employed existing sampling-based methods [12] to solve discretized instances of the GTSPN using the centers of regions and sampled regions with up to $k = 100$ samples denoted as Centroid-TSP and Centroid-GTSP, respectively.

The results on the solution value $\mathcal{L}$ are summarized in Fig. 2, where the lower bound values $\mathcal{LB}$ are obtained by the proposed $\mathrm{BnB}_{\mathrm{Ipopt}}$ solver. The results indicate that the proposed $\mathrm{BnB}_{\mathrm{Ipopt}}$ provides tight lower bound values on the optimal solution that supports the feasibility of the proposed approach using the NLP model.

III. Conclusion

We report on the developed BnB-based solution to the robotic multi-goal planning motivated by visual inspection tasks with a mobile robotic manipulator, which represents a complex robotic system. We propose to address the robot's motion constraints using the NLP model of the robot's kinematics. Even though locally optimal solver `Ipopt` has been employed, the reported results support the feasibility of the proposed approach that provides high-quality solutions close to the obtained lower bound values on the optimal solution cost. The results motivate further study of modeling robots' motion constraints as programming models.

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